The Minnesota Journal of Undergraduate Mathematics

Probabilistic Analysis of the Economic Impact of Earthquake Prediction Systems

Tiffany Kolba and Ruyue Yuan

Valparaiso University

The Minnesota Journal of Undergraduate Mathematics

Volume 1 Number 1 (2015)

Sponsored by
School of Mathematics
University of Minnesota
Minneapolis, MN 55455

The Minnesota Journal of Undergraduate Mathematics

Volume 1 Number 1 (2015)

Probabilistic Analysis of the Economic Impact of Earthquake Prediction Systems

Tiffany Kolba and Ruyue Yuan

Valparaiso University

Abstract. In order to study the economic impact of an earthquake prediction system, we use probabilistic methods to model the expected cost per life saved from a prediction system. We improve upon previous work by directly modeling the expected cost per life saved, $E[C/L]$, rather than the ratio of the expected cost to the expected number of lives saved, $E[C]/E[L]$. The latter is shown to always be an underestimate of the former. The model is applied numerically to the San Francisco Bay area and the expected cost per life saved from an earthquake prediction system over a 50 year period is found to be \$3.3 million. While the amount is quite high, it is substantially lower than the corresponding expected cost per life saved of \$6.3 million from expenditures in earthquake engineering to improve building codes. Therefore, we conclude that earthquake prediction systems provide a valuable public good.

1. Introduction

This work is motivated by the question of whether or not earthquake prediction systems are actually worth investing in, as the cost of operating such systems is quite large compared to the number of lives which may be saved and, furthermore, false predictions may cause large-scale public panics and substantial economic losses. Some argue that it is more effective to just invest in the research and development of infrastructure which can withstand earthquakes, rather than trying to predict earthquakes before they occur [2].

We build upon Paté's research [4] in order to evaluate the economic impact of an earthquake prediction system. Paté used the expected cost per life saved from an earthquake prediction system as a measure of the economic impact. The cost of an earthquake prediction system is modeled as a net of direct costs of the prediction system and indirect costs from false predictions, minus the costs which are prevented from correct predictions. The direct costs include the research, development, and implementation of the prediction system, as well as the annual cost of operation of the system. The indirect costs from a false prediction include the loss of labor force due to migration, the loss of capital, and the temporary decrease in consumption, as well as the cost of protective measures taken by individuals, businesses, and the government. The costs which are prevented from correct predictions include avoided property damage and avoided loss of economic production. The total net cost is computed over a 50 year period.

However, Paté approximated the expected cost per life saved from an earthquake prediction system by the ratio of the expected cost to the expected number of lives saved. We improve upon Paté's research by directly modeling the expected cost per life saved, and we show in Theorem 1 that Paté's approximation underestimates the true expected cost per life saved. Furthermore, Paté modeled the cost and lives saved as deterministic functions of the occurring magnitude, predicted magnitude, and lead time of the prediction. Instead, we model the cost and lives saved as random functions of the occurring magnitude, predicted magnitude, and lead time of the prediction, which is a more realistic model of the underlying processes.

2. Probabilistic Model

In 1935, Charles Richter developed a logarithmic scale to measure the quantity of energy released by an earthquake. However, the Richter scale was replaced in the 1970s with the Moment Magnitude Scale, which is also logarithmic and similar to the Richter scale, but is more accurate for measuring large earthquakes [1]. The Moment Magnitude Scale is the standard used by the United States Geological Survey, and all earthquake magnitudes in this paper will be assumed to be measured in the Moment Magnitude Scale. Although the scale for earthquake magnitudes is continuous, we discretize the range into six categories with 0-4 as one category since these magnitudes are too small to cause damage or loss of life. Hence, we define the following discrete random variables:

M_O = Magnitude of occurring earthquake, with possible values in {0-4, 4-5, 5-6, 6-7, 7-8, 8+}

M_P = Magnitude of predicted earthquake, with possible values in {4-5, 5-6, 6-7, 7-8, 8+}

T = Lead time between the prediction and the occurring earthquake, with possible values short (order of a day), medium (order of a month), and long (order of a year).

For the predicted magnitude, M_P, we exclude the category 0-4 since a magnitude in this range would be unnecessary to predict. We let $p(m_O, m_P, t)$ denote the joint probability mass function of M_O, M_P, and T. The joint probability mass function has the property that

$$\begin{aligned} p(m_O, m_P, t) &= p(m_O) \cdot p(m_P, t | m_O) \\ &= p(m_O) \cdot p(m_P | m_O) \cdot p(t | m_O, m_P) \\ &= p(m_O) \cdot p(m_P | m_O) \cdot p(t | m_P) \end{aligned} \quad (1)$$

where the last equality comes from Paté's assumption that given the occurring magnitude and the predicted magnitude, the lead time only depends upon the predicted magnitude. The three terms in (1) can be approximated using data for the geographical region of interest.

We let C denote the cost of an earthquake prediction system and L denote the number of lives saved. The cost, C, is actually a net cost satisfying $C = C_P - C_A$, where C_P is the direct and indirect costs of the prediction system and C_A is the avoided cost from a correct prediction. In Paté's work, C_P, C_A, and L are modeled as deterministic functions of M_O, M_P, and T. However, we model C_P, C_A, and L to be random functions of M_O, M_P, and T. We utilize a stochastic model because not every earthquake with a given magnitude and given prediction parameters will have the same exact cost and number of lives saved. Thus, we model C_P, C_A, and L with the following probability distributions:

$$C_P \sim Normal(\mu_P, \sigma_P) \tag{2}$$

where μ_P and σ_P are deterministic functions of M_P and T,

$$C_A \sim Exp(\alpha\beta) \tag{3}$$

where β is the mean cost of damages after an earthquake occurs if there were no earthquake prediction system and hence is a deterministic function of solely M_O, and α represents the proportion of damages avoided due to the prediction system and is a deterministic function of M_O, M_P, and T, and

$$L = \tilde{L} + 1, \quad \tilde{L} \sim Poisson(\gamma\lambda) \tag{4}$$

where λ is the mean number of casualties after an earthquake occurs if there were no earthquake prediction system and hence is a deterministic function of solely M_O, and γ represents the proportion of casualties avoided due to the prediction system and is a deterministic function of M_O, M_P, and T.

The cost of the prediction system, C_P, is modeled as a continuous random variable following a Normal distribution since we expect this cost to be fairly concentrated symmetrically about the mean value. On the other hand, the avoided cost from a correct prediction, C_A, is modeled as a continuous random variable following an Exponential distribution because we expect the cost of damages following an earthquake to have a distribution for which the probability decays rapidly for large values. Note that we model the number of lives saved, L, as a discrete random variable with a minimum value of one since if there are no lives saved, the cost per life saved is infinite.

3. Cost Per Life Saved

In Paté's work, the expected cost per life saved from an earthquake prediction system is approximated by the ratio of the expected cost to the expected number of lives saved. However, in Section 3.1 we directly compute the expected cost per life saved using the probability model defined in the previous section and then in Section 3.2 we show that Paté's approximation underestimates the true value. In, Section 3.3 we also give bounds on the variance of the cost per life saved.

3.1. **Expected Cost Per Life Saved.** By properties of conditional expectation [7],

$$E\left[\frac{C}{L}\right] = E\left[E\left[\frac{C}{L}\middle| M_O, M_P, T\right]\right].$$

We now apply the assumption that given M_O, M_P, and T, the cost and number of lives saved are independent. Note that this does not imply that the cost and number of lives saved are independent, but rather that the dependence between the cost and number of lives saved is solely through the values of M_O, M_P, and T. Hence,

$$E\left[\frac{C}{L}\right] = E\left[E\left[C|M_O, M_P, T\right] \cdot E\left[\frac{1}{L}\middle|M_O, M_P, T\right]\right].$$

Now since the number of lives saved, $L = \tilde{L} + 1$, with $\tilde{L} \sim Poisson(\gamma\lambda)$,

$$\begin{aligned} E\left[\frac{1}{L}\middle|M_O, M_P, T\right] &= E\left[\frac{1}{\tilde{L}+1}\middle|M_O, M_P, T\right] \\ &= \sum_{k=0}^{\infty} \frac{1}{k+1} \cdot \frac{e^{-\gamma\lambda} \cdot (\gamma\lambda)^k}{k!} = \frac{1 - e^{-\gamma\lambda}}{\gamma\lambda} \end{aligned}$$

where λ is a deterministic function of solely M_O and γ is a deterministic function of M_O, M_P, and T. Note that this expression holds for $\gamma\lambda > 0$. When $\gamma\lambda = 0$, which it will in the case of a small earthquake with M_O in the range of 0-4 or in the case where there is no prediction, $E\left[\frac{1}{L}|M_O, M_P, T\right] = 1$.

Since the net cost $C = C_P - C_A$ with $C_P \sim Normal(\mu_P, \sigma_P)$ and $C_A \sim Exp(\alpha\beta)$,

$$E\left[C|M_O, M_P, T\right] = \mu_p - \alpha\beta$$

where μ_P is a deterministic function of M_P and T, α is a deterministic function of M_O, M_P, and T, and β is a deterministic function of solely M_O. Thus, the expected value of the cost per life saved is

$$\begin{aligned} E\left[\frac{C}{L}\right] =& E\left[(\mu_P - \alpha\beta)\left(\frac{1 - e^{-\gamma\lambda}}{\gamma\lambda}\right)\right] \\ =& \sum_{t}\sum_{m_P}\sum_{m_O}\Big[(\mu_P(m_P, t) - \alpha(m_O, m_P, t)\beta(m_O)) \\ & \cdot \frac{1 - e^{-\gamma(m_O, m_P, t)\lambda(m_O)}}{\gamma(m_O, m_P, t)\lambda(m_O)} \cdot p(m_O) \cdot p(m_P|m_O) \cdot p(t|m_P)\Big] \end{aligned} \quad (5)$$

where the dependencies upon M_O, M_P, and T have now been denoted explicitly. Since there are 6 possibilities for the value of M_O, 5 possibilities for M_P, and 3 for T, in order to obtain a numerical value for the expected cost per life saved for a particular geographical region of interest, α and γ would need to be numerically approximated for all 90 combinations of M_O, M_P, and T, as well as μ_P for all 15 combinations of M_P and T, and β and λ for all 6 possibilities of M_O. As mentioned previously, we would also need to numerically approximate $p(m_O)$ for the 6 possible values of M_O, $p(m_P|m_O)$ for the 30 combinations of M_O and M_P, and $p(t|m_P)$ for the 15 combinations of M_P and T.

3.2. **Comparison to Previous Work.** Paté's approximation to the expected cost per life saved with the ratio of the expected cost to the expected number of lives saved, i.e.,

$$E\left[\frac{C}{L}\right] \approx \frac{E[C]}{E[L]},$$

contains two errors. Firstly, the cost and number of lives saved are not independent, and hence

$$E\left[\frac{C}{L}\right] \neq E[C]\cdot E\left[\frac{1}{L}\right].$$

The second error in the approximation is due to the fact that

$$E\left[\frac{1}{L}\right] \neq \frac{1}{E[L]}.$$

In the following theorem, we show that the ratio of the expected cost to the expected number of lives saved is in fact an underestimate to the true expected cost per life saved.

Theorem 1. The expected cost per life saved from an earthquake prediction system is always greater than or equal to the ratio of the expected cost to the expected number of lives saved; i.e.,

$$E\left[\frac{C}{L}\right] \geq \frac{E[C]}{E[L]}.$$

Proof. By properties of covariance [7],

$$E\left[\frac{C}{L}\right] = E[C]\cdot E\left[\frac{1}{L}\right] + Cov\left(C, \frac{1}{L}\right) \tag{6}$$

Since $\frac{1}{L}$ is a convex function, by Jensen's Inequality [6],

$$E\left[\frac{1}{L}\right] \geq \frac{1}{E[L]}. \tag{7}$$

Now when more lives are saved, there is also the tendency to have more avoided damages, meaning that C is smaller when L is larger, which in turn implies that

$$Cov\left(C, \frac{1}{L}\right) \geq 0. \tag{8}$$

Combining (6), (7), and (8), we obtain the desired result. □

Since the expected cost per life saved can be utilized in public policy decision making, an underestimate of the cost could have significant consequences, while our model of the expected cost per life saved can help give a more accurate assessment of the true economic impact of earthquake prediction systems.

3.3. **Variance of Cost Per Life Saved.** Since there is no guarantee that the actual cost per life saved from an earthquake prediction system is close to the expected value, it is helpful to have an idea of the variance of the cost per life saved. By properties of variance [7],

$$V\left(\frac{C}{L}\right) = E\left[\left(\frac{C}{L}\right)^2\right] - \left(E\left[\frac{C}{L}\right]\right)^2.$$

Since we already modeled $E[\frac{C}{L}]$ in Section 3.1, it only remains to compute $E[(\frac{C}{L})^2]$. By our previous assumption that C and L are independent given M_O, M_P, T, we have that

$$\begin{aligned} E\left[\left(\frac{C}{L}\right)^2\right] =& E\left[E\left[\frac{C^2}{L^2}\middle| M_O, M_P, T\right]\right] \\ =& E\left[E\left[C^2|M_O, M_P, T\right] \cdot E\left[\frac{1}{L^2}|M_O, M_P, T\right]\right]. \end{aligned} \tag{9}$$

Using our probabilistic model for C given in equations (2) and (3),

$$E[C^2|M_O, M_P, T] = (\mu_P - \alpha\beta)^2 + (\sigma_P)^2 + (\alpha\beta)^2. \tag{10}$$

Using our probabilistic model for L given in equation (4),

$$E\left[\frac{1}{L^2}\middle| M_O, M_P, T\right] = \sum_{k=0}^{\infty} \frac{1}{(k+1)^2} \cdot \frac{e^{-\gamma\lambda} \cdot \gamma\lambda^k}{k!}.$$

While this sum cannot be computed explicitly in closed form, we can give the following lower and upper bounds:

$$\sum_{k=0}^{\infty} \frac{e^{-\gamma\lambda} \cdot (\gamma\lambda)^k}{(k+2)!} \leq E\left[\frac{1}{L^2}\middle| M_O, M_P, T\right] \leq \sum_{k=0}^{\infty} 2 \cdot \frac{e^{-\gamma\lambda} \cdot (\gamma\lambda)^k}{(k+2)!}.$$

The summations for the lower and upper bounds can now be computed explicitly, in order to obtain the following bounds:

$$\frac{1 - e^{-\gamma\lambda} - \gamma\lambda e^{-\gamma\lambda}}{(\gamma\lambda)^2} \leq E\left[\frac{1}{L^2}\middle| M_O, M_P, T\right] \leq \frac{2 - 2e^{-\gamma\lambda} - 2\gamma\lambda e^{-\gamma\lambda}}{(\gamma\lambda)^2}. \tag{11}$$

Note that these expressions for the lower and upper bounds hold for $\gamma\lambda > 0$. When $\gamma\lambda = 0$, as it does in the case of a small earthquake with magnitude in the range 0-4 or in the case of no prediction, $E[\frac{1}{L^2}|M_O, M_P, T] = 1$, which is the limiting value of the upper bound as $\gamma\lambda \to 0$. Also note that Jensen's Inequality gives the lower bound

$$E\left[\frac{1}{L^2}\middle| M_O, M_P, T\right] \geq \frac{1}{(E[L|M_O, M_P, T])^2} = \frac{1}{(\gamma\lambda + 1)^2}$$

but the lower bound given in (11) is larger, and hence more useful, for all $\gamma\lambda \geq 1$. By plugging in the expression for the conditional expectation of C^2 given in (10) and the bounds for the conditional expectation of $\frac{1}{L^2}$ given in (11) into the formula for $E[(\frac{C}{L})^2]$ given in (9), we can obtain bounds for $E[(\frac{C}{L})^2]$, which in turn will result in bounds for the variance of the expected cost per life saved from an earthquake prediction system. In order to obtain numerical values for the bounds on the variance, the same parameters as in Section 3.1 would need to be numerically approximated, as well as σ_P for all 15 combinations of possible values for M_P and T.

4. Numerical Example: San Francisco Bay Area

We now apply our model for the expected cost per life saved to the San Francisco Bay Area, which is a highly seismic zone between the San Andreas fault and the Hayward-Calaveras fault [4]. We use the same numerical estimates for μ_P, α, β, γ, λ, and the joint

	m_O	0-4	4-5	5-6	6-7	7-8	8+
Probability	$p(m_O)$	0.5	0.2	0.15	0.1	0.042	0.008
Initial Mean Damages	$\beta(m_O)$	0	2	50	500	10,000	25,000
Initial Mean Casualties	$\lambda(m_O)$	0	1	10	200	4,000	15,000

Table 1. The probability that the magnitude of an occurring earthquake is in the specified range, as well as the mean damages in millions of dollars and the mean number of casualties if there were no earthquake prediction system, for each magnitude range, in a given year for the San Francisco Bay Area.

probability mass function of M_O, M_P, and T as calculated by Paté in [3]. Note that for some of the parameters, Paté gave estimates for a "low case," "base case," and "high case" scenario. We use the "base case" estimates for all of our numerical approximations. Since in Paté's work, the cost and number of lives saved are deterministic functions of M_O, M_P, and T, the three different cases were used for sensitivity analysis. However, since in our model the cost and number of lives saved are random functions of M_O, M_P, and T, we utilize the "low case" and "high case" estimates to approximate the additional parameter needed for the variance of the cost per life saved, namely σ_P.

Table 1 lists the estimates for $p(m_O)$, the probability that an occurring earthquake has a specified magnitude, as well as $\beta(m_O)$ and $\lambda(m_O)$, the mean damages in millions of dollars and mean number of casualties, respectively, for each occurring magnitude range in the case of no earthquake prediction system. These estimates are taken directly from Table 60 of [3]. The estimates of the other parameters can be found in Tables 61-70 of [3]. Plugging all of the numerical approximations into the model for the expected cost per life saved given in (5), we obtain an expected cost per life saved of \$136.7 million. From the bounds on the variance given in Section 3.3, we obtain a range for the standard deviation of \$521.4 million to \$570.4 million. Thus, the cost per life saved has an extremely high expected value with an even higher degree of variability.

Computing the expected cost and expected number of lives saved separately, we obtain a total expected cost of \$173.8 million and a total expected number of lives saved of 228, giving a ratio of \$0.76 million. The ratio of \$0.76 million drastically underestimates the true expected cost per life saved of \$136.7 million mainly because of the lack of independence between the cost and lives saved. The cost is highest when there are few lives saved and smallest when there are many lives saved due to more prevented damages; hence, the true expected cost per life saved is much higher than the ratio of expected values due to the high cost per life saved for small earthquakes or no predictions. In fact, based upon the estimated parameters, there is a 53.6% chance of having no lives saved due to either an occurring earthquake with magnitude 0-4, which has no loss of life, or an earthquake with magnitude above 4, but no prediction and hence no avoided casualties.

Now the expected cost per life saved of \$136.7 million is calculated over a period of only one year, so in order to have a more robust idea of the expected cost per life saved due to an earthquake prediction system, it is helpful to consider the expected cost per life saved over a period of N years. Following Paté's work, we compute the expected cost per life saved over a period of 50 years. We let $C = C_1 + C_2 + ... + C_{50}$ where C_i is the cost for year

i and all the C_i's are independent, identically distributed random variables following the probability model outlined in Section 2. Similarly, we let $L = L_1 + L_2 + ... + L_{50}$ where L_i is the number of lives saved for year i and all the L_i's are independent, identically distributed random variables following the probability model outlined in Section 2. The advantage of Paté's approximation is that the ratio of the expected cost to the expected number of lives saved over a 50 year period is equal to simply the ratio of the expected cost to the expected number of lives saved for one year; i.e.,

$$\frac{E[C_1 + C_2 + ... + C_{50}]}{E[L_1 + L_2 + ... + L_{50}]} = \frac{50 \cdot E[C_i]}{50 \cdot E[L_i]} = \frac{E[C_i]}{E[L_i]}.$$

As for the true expected cost per life saved over a 50 year period, we have the following inequality:

$$\frac{E[C_i]}{E[L_i]} \leq E\left[\frac{C_1 + C_2 + ... + C_{50}}{L_1 + L_2 + ... + L_{50}}\right] \leq E\left[\frac{C_i}{L_i}\right].$$

Thus, the true expected cost per life saved over a 50 year period is somewhere in between \$.76 million and \$136.7 million. Exact calculation of the true expected value would require summing over the joint probability mass function of the 50 years, which is computationally intractable. Hence, we employ Monte Carlo simulation [5] to obtain an estimate of \$3.3 million for the expected cost per life saved from an earthquake prediction system over a 50 year period in the San Francisco Bay Area.

Based upon constraints of available data, all of the estimates used were calculated in "1978 dollars" for a 50 year period from 1978-2028 assuming constant \$30 million annual funding for earthquake prediction system operating costs, constant building codes, constant level of prediction accuracy, fixed probability distribution of an occurring earthquake's magnitude, no inflation, and no population or economic growth. In [4], Paté also assumed flat annual funding, constant building codes, fixed magnitude probabilities, and no inflation, but did take into account increased prediction accuracy over the 50 year period due to advances in seismology, as well as a 2% growth rate and a 7% social rate of discount, obtaining a total expected cost of \$4.6 billion and an expected 2943 lives saved, producing a ratio of \$1.56 million for the expected cost to the expected number of lives saved over the 50 year period. While this is larger that the ratio of expected values we calculated of \$.76 million, it still substantially underestimates the true expected cost per life saved. Note that taking into account these additional measures would only have increased our estimate of \$3.3 million, especially since we utilized the prediction probabilities from the end of Paté's 50 year period where the accuracy is highest. Taking into account lower accuracy during the beginning of the 50 year period increases the cost per life saved because of high expenditures due to false predictions.

In [4], Paté also approximated the expected cost per life saved from expenditures in earthquake engineering to improve building codes over the 50 year period from 1978-2028 and obtained an estimate of \$6.3 million. Comparing this approximation to her approximation of \$1.56 million for the expected cost per life saved from an earthquake prediction system, Paté concluded that earthquake prediction systems are economically competitive and worth investing in. Now the approximation to the expected cost per life saved from improved building codes again made the error of using the ratio of the expected cost to the expected number of lives saved (the expected cost was \$2.58 billion with an expected number of lives saved of 412). However, by the same reasoning as in Theorem 1, the ratio of expected values underestimates the true expected cost per life saved from improved building codes. Thus, we can safely conclude that our estimate for the expected cost per life saved from an earthquake prediction system of $3.3 million is smaller than the corresponding value for improved building codes.

Since we are nearing the end of this 50 year period, future work could compare the actual cost per life saved from earthquake prediction systems in the San Francisco Bay Area from 1978-2028 to the expected value computed here, as well as to the actual cost per life saved from improved building codes. In addition, future work could update the estimates for the parameters in the model based upon current levels of funding, current building codes, advances in earthquake prediction accuracy, and demographic and economic changes in the area, as well as adjust the model for inflation.

5. Conclusion

With limited budgets, government agencies are faced with hard decisions about which sectors and projects to prioritize funding in order to achieve the desired goals. When it comes to protecting the public from earthquake hazards, the two principal approaches government agencies can pursue are earthquake prediction and improved building codes. The expected cost per life saved is a useful measurement for evaluating various approaches when making public policy decisions. We improved upon previous work by directly modeling the expected cost per life saved from an earthquake prediction system, rather than using the ratio of the expected cost to the expected number of lives saved, which we showed is always an underestimate of the true value. In addition, while previous work modeled the cost and number of lives saved as deterministic functions of the occurring earthquake's magnitude, predicted magnitude, and lead time of the prediction, we more realistically modeled the cost and number of lives saved as random functions.

We applied our model to the San Francisco Bay Area and obtained an expected cost per life saved of $3.3 million over the 50 year period from 1978-2028. While the amount is quite high, it appears to be substantially lower than the corresponding expected cost per life saved from expenditures in earthquake engineering to improve building codes. Therefore, we conclude that earthquake prediction systems provide a valuable public good, at least in the San Francisco Bay Area. Future work could compare the expected cost per life saved from an earthquake prediction system across various geographic regions.

Furthermore, since our proof that the ratio of the expected cost to the expected number of lives saved underestimates the true value likely applies to many other sectors in addition to earthquake prediction systems, we hope that our model here will encourage future work with cost-benefit analyses to directly model the expected cost per life saved. While the expected cost per life saved requires more sophisticated probabilistic analysis than the ratio of the expected values, having a more accurate assessment of the true value is important for making sound public policy decisions.

References

[1] *USGS Earthquake Magnitude Policy*, January 2002.

[2] J. Eugene Haas. Earthquake prediction: Is it better not to know? *MOSAIC*, 8(2):8–14, March 1977.

[3] Marie-Elisabeth Pate. *Public Policy in Earhquake Effects Mitigation: Earthquake Engineering and Earthquake Prediction*. PhD thesis, Stanford University, May 1978.

[4] Marie-Elisabeth Pate and Haresh C. Shah. Public policy issues: Earthquake prediction. *Bulletin of the Seismological Society of America*, 69(5):1533–1547, October 1979.

[5] Christian Robert and George Casella. *Monte Carlo Statistical Methods*. Springer, 2nd edition, 2005.

[6] Sheldon Ross. *A First Course in Probability*. Pearson, 9th edition, 2012.

[7] Dennis D. Wackerly, William Mendenhall, and Richard L. Scheaffer. *Mathematical Statistics with Applications*. Cengage Learning, 7th edition, 2008.

Student biographies

Ruyue Yuan: (Ruyue.Yuan@valpo.edu) Ruyue Yuan graduated from Valparaiso University in May 2015 with a B.S. in mathematics and minors in applied statistics and honors humanities. She completed an REU in combinatorics at East Tennessee State University during Summer 2013 and participated in the Budapest Semester in Mathematics during Fall 2014. She is currently a PhD student in the Department of Mathematics at the University of Florida.